



Fluid Dynamics and Transport Phenomena

Diffusion of chemically reactive species in third grade fluid flow over an exponentially stretching sheet considering magnetic field effects

T. Hayat^{1,2}, M. Ijaz Khan¹, M. Waqas¹, A. Alsaedi², T. Yasmeen^{3,4,*}¹ Department of Mathematics, Quaid-I-Azam University, 45320, Islamabad 44000, Pakistan² Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, P. O. Box 80257, Jeddah 21589, Saudi Arabia³ Department of Mechanical Engineering, Imperial College London, London SW7 2AZ, UK⁴ Department of Mechanical Engineering, University of Engineering & Technology Peshawar, Pakistan

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ABSTRACT

This article addresses the magnetohydrodynamics (MHD) flow of a third grade fluid over an exponentially stretching sheet. Analysis is carried out in the presence of first order chemical reaction. Both cases of constructive and destructive chemical reactions are reported. Convergent solutions of the resulting differential systems are presented in series forms. Characteristics of various sundry parameters on the velocity, concentration, skin friction and local Sherwood number are analyzed and discussed.

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1. Introduction

There are several materials like shampoos, mud, soaps, apple sauce, sugar solution, polymeric liquids, tomato paste, condensed milk, paints, and blood at low shear rate which show the characteristics of non-Newtonian fluids. The behavior of such materials cannot be explored by a single constitutive relationship because of their diverse properties. Hence different fluid models are developed in the past to describe the exact nature of non-Newtonian materials. Third grade fluid is a subclass of differential type non-Newtonian fluid. This fluid model exhibits shear thickening and shear thinning characteristics. Some studies on the third grade fluid can be seen in refs. [1–5]. Further, the flow over a stretching surface is an important problem in many engineering processes with applications in various engineering and industrial processes like cooling of metallic sheets in a cooling bath, annealing and thinning of copper wires, aerodynamic extrusion of plastic and rubber sheets, drawing of plastic films and sheets, glass fiber and paper production etc. It is worth mentioning to point out here that the stretching velocity is not linear necessarily in all the cases. The stretching velocity may be nonlinear or exponential. For example in annealing and thinning of copper wires, the desired quality product depends on the continuous stretching of surface with exponential dependence velocity distribution [6–10].

It is well known fact that the raw materials are constructed to undergo chemical reaction in industrial chemical processes to convert cheaper raw materials into higher standard products. Such chemical transformations are performed in a reactor. The reactor has a key role to providing an appropriate environment for suitable time and allowing the removal of finished products. The study of chemical reaction has important role in chemical technologies like polymer production. No doubt a chemical reaction can be classified either through homogeneous or heterogeneous processes. Reaction rate in first order chemical reaction is directly proportional to the concentration. A large amount of research work has been reported in this field. For instance, Soret and Dufour effects in three-dimensional flow over an exponentially stretching surface with porous medium, chemical reaction and heat source/sink is studied by Hayat *et al.* [11]. Bhattacharyya and Layek [12,13] addressed the behavior of chemically reactive solute distribution in MHD boundary layer flow over a permeable stretching sheet and also described the slip effects on the boundary layer flow and mass transfer over a vertical stretching sheet. MHD stagnation point flow and heat transfer impinging on stretching sheet with chemical reaction and transpiration is examined by Mabood *et al.* [14]. Sheikh and Abbas [15] explored the effects of thermophoresis and heat generation/absorption on MHD flow due to an oscillatory stretching sheet with chemically reactive species. Convective flow of micropolar liquid with chemical reaction and mixed convection is examined by Swapna *et al.* [16]. Hayat *et al.* [17] reported carbon nanotubes effects in the stagnation point flow towards a nonlinear stretching sheet with homogeneous-heterogeneous reactions. Characteristics of thermal

* Corresponding author at: Department of Mechanical Engineering, Imperial College London, London SW7 2AZ, UK.

E-mail addresses: mw_qau88@yahoo.com (M. Waqas),

tabassum.mechanical@gmail.com, t.yasmeen16@imperial.ac.uk (T. Yasmeen).

radiation and chemical reaction in flow of nanofluid saturating porous medium is presented by Zhang *et al.* [18]. Mythili and Sivaraj [19] reported the chemically reactive flow of Casson liquid towards a vertical cone. Narayana and Babu [20] analyzed thermal radiation and chemical reaction effects in flow of Jeffrey liquid over a stretched surface. Impact of Cattaneo–Christov heat flux model in flow of variable thermal conductivity fluid over a variable thicked surface with chemical reaction is reported by Hayat *et al.* [21]. Hayat *et al.* [22] explored stagnation point flow with Cattaneo–Christov heat flux and homogeneous–heterogeneous reactions. Homogeneous–heterogeneous reactions in MHD flow due to an unsteady curved stretching surface are scrutinized by Imtiaz *et al.* [23]. Hayat *et al.* [24] studied the impact of Cattaneo–Christov heat flux in Jeffrey fluid flow with homogeneous–heterogeneous reactions. Influences of thermal radiation and chemical reaction in MHD flow by a cylinder are explored by Machireddy [25]. Flow due to nonlinear stretching surface with chemical reaction and porous medium is addressed by Ziaabaksh *et al.* [26]. Bhattacharya [27] developed dual solutions for stagnation point flow past a stretching/shrinking sheet with chemical reaction. Chen and Sun [28] addressed boundary layer interaction of chemical reacting flow in shock tube. Influence of chemical reaction and radiation in MHD free convective flow are addressed by Raju *et al.* [29].

The study of magnetohydrodynamics (MHD) flow of an electrically conducting fluid over a stretching sheet has promising applications in modern metallurgical as well as in metal-working procedures. Many professional techniques regarding polymers require the cooling of continuous strips and filaments by drawing them from moving fluid. The final product depends greatly on the rate of cooling that is governed by the structure of the boundary layer close to the stretching sheet. Mukhopadhyay *et al.* [30] studied MHD flow of Casson fluid due to exponentially stretching sheet with thermal radiation. Impact of magnetohydrodynamics in bidirectional flow of nanofluid subject to second order slip velocity and homogeneous–heterogeneous reactions is reported by Hayat *et al.* [31]. Lin *et al.* [32] examined unsteady MHD pseudo-plastic nanofluid flow and heat transfer in a finite thin film over stretching surface with internal heat generation. Sheikholeslami *et al.* [33] analyzed the effect of thermal radiation on magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model. Application of the HAM-based Mathematica package BVP4c 2.0 on MHD Falkner–Skan flow of nanofluid is provided by Farooq *et al.* [34]. Shehzad *et al.* [35] presented an analytical study to investigate thermal radiation effects in three-dimensional flow of Jeffrey nanofluid with internal heat generation and magnetic field.

Here our main theme is to study the influences of magnetohydrodynamics (MHD) flow of third grade fluid by an exponentially stretching sheet. Mass transfer analysis is performed in the presence of first order chemical reaction. The governing nonlinear flow model is solved and homotopic solutions [36–53] of dimensionless velocity and concentration are presented. Physical quantities for various parameters of interest are examined. To our knowledge such analysis is not yet reported.

2. Formulation

Consider the two-dimensional hydromagnetic flow of third grade fluid over an exponentially stretching sheet. Mass transfer effects are taken into account in the presence of chemical reaction. An applied magnetic field of strength B_0 is encountered normal to the flow direction. The magnetic Reynolds number is chosen small. Further the induced magnetic field is smaller in comparison to the applied magnetic field and is negligible. The two-dimensional boundary layer equations of an incompressible third grade fluid with mass transfer are [5,7]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho} \left(u \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + 3 \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right) + 2 \frac{\alpha_2}{\rho} \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + 6 \frac{\beta_3}{\rho} \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u, \quad (2)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k(C - C_\infty), \quad (3)$$

subject to the boundary conditions [7]

$$u = U_w(x) = U_0 \exp\left(\frac{x}{l}\right), v = 0, C = C_\infty + C_0 \exp\left(\frac{x}{2l}\right) \text{ at } y = 0, \\ u \rightarrow 0, C \rightarrow C_\infty \text{ as } y \rightarrow \infty. \quad (4)$$

Here u and v denote the velocity components in the x and y directions respectively, ν the kinematic viscosity, α_1 , α_2 and β_3 the fluid parameters, ρ the fluid density, σ the electric charge density, D the mass diffusion, (U_0, C_0) the reference velocity and concentration respectively, (C, C_∞) the fluid and ambient concentrations respectively and $k = k_0 \exp(\frac{x}{l})$ the exponential reaction rate, where $k > 0$ stands for destructive reaction whereas $k < 0$ stands for constructive reaction, k_0 is a constant.

Introducing the similarity variables as:

$$\eta = \sqrt{\frac{U_0}{2\nu l}} \exp\left(\frac{x}{2l}\right) y, \psi = \sqrt{2\nu l U_0} f(\eta) \exp\left(\frac{x}{2l}\right), u = U_0 \exp\left(\frac{x}{l}\right) f'(\eta), \\ v = -\sqrt{\frac{\nu U_0}{2l}} \exp\left(\frac{x}{2l}\right) [f(\eta) + \eta f'(\eta)], \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \quad (5)$$

Here ψ denotes the stream function, f the dimensionless velocity and ϕ the dimensionless concentration. Employing above transformations, Eq. (1) is identically satisfied while Eqs. (2)–(4) are

$$f''' - 2(f')^2 + ff'' + \alpha_1 \left(3f'f''' - ff^{iv} - 2\eta f''f''' - 9(f'')^2 \right) - \alpha_2 \left(3(f'')^2 + \eta f'f''' \right) + 3\beta_3 (f'')^2 f''' - 2Ha^2 f' = 0, \quad (6)$$

$$f(0) = 0, f'(0) = 1, f'(\infty) = 0, \quad (7)$$

$$\phi'' + Sc(f\phi' - f'\phi - 2\gamma\phi) = 0, \quad (8)$$

$$\phi(0) = 1, \phi(\infty) = 0, \quad (9)$$

here α_1 , α_2 and β_3 denote the fluid parameters, Ha the Hartman number, Sc the Schmidt number and γ the chemical reaction parameter. It is noted that $\gamma > 0$ is for destructive chemical reaction, $\gamma < 0$ for generative chemical reaction and $\gamma = 0$ corresponds to non-reactive species. These quantities are defined as

$$\alpha_1 = \frac{U_0 \alpha_1^* \exp(\frac{x}{l})}{\nu \rho l}, \alpha_2 = \frac{U_0 \alpha_2^* \exp(\frac{x}{l})}{\nu \rho l}, \beta_3 = \frac{U_0^3 \beta_3^* \exp(\frac{3x}{l})}{\nu^2 \rho l}, \\ Ha^2 = \frac{\sigma B_0^2 l}{\rho U_w(x)}, Sc = \frac{\nu}{D}, \gamma = \frac{k_0 l}{U_0}. \quad (10)$$

The skin friction coefficient C_f and local Sherwood number Sh_x are

$$C_f = \frac{\tau_w}{\rho U_w^2}, Sh_x = \frac{x j_w}{D(C_w - C_\infty)}, \quad (11)$$

with

$$\tau_w = \left[\mu_0 \frac{\partial u}{\partial y} + \frac{\alpha_1^*}{\rho} \left(2 \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \nu \frac{\partial^2 u}{\partial y^2} + u \frac{\partial^2 u}{\partial x \partial y} \right) + 2 \frac{\beta_3}{\rho} \left(\frac{\partial u}{\partial y} \right)^3 \right]_{y=0}, \quad (12)$$

$$j_w = -D \left(\frac{\partial C}{\partial y} \right)_{y=0}.$$

Dimensionless expressions of skin friction coefficient and local Sherwood number are

$$\frac{1}{\sqrt{2}} \sqrt{Re_x} C_f = \left(\left[f''(\eta) + \alpha_1 \left(\frac{7}{2} f'(\eta) f''(\eta) - \frac{1}{2} f f''' \right) + \beta (f''(\eta)^3) \right] \right)_{\eta=0},$$

$$Sh/Re_x^{1/2} = -\sqrt{\frac{X}{2}} \phi'(0), \quad (13)$$

with $X = \frac{x}{\nu}$, $Re_x = U_w(x)x/\nu$.

3. Series Solutions

In order to obtain analytical solution, the selected initial guesses and auxiliary linear operators are:

$$f_0(\eta) = 1 - e^{-\eta}, \phi_0(\eta) = e^{-\eta}, \quad (14)$$

$$L_f = f''' - f', L_\phi = \phi'' - \phi. \quad (15)$$

The above auxiliary linear operators satisfy the following properties

$$L_f(C_1 + C_2 e^\eta + C_3 e^{-\eta}) = 0, \quad (16)$$

$$L_\phi(C_4 e^\eta + C_5 e^{-\eta}) = 0, \quad (17)$$

where C_i ($i = 1 - 5$) indicates the arbitrary constants.

4. Zeroth-Order Deformation Problem

The corresponding problems at the zeroth order are:

$$(1-p)L_f[\hat{f}(\eta; p) - f_0(\eta)] = p h_f \mathbf{N}_f[\hat{f}(\eta; p), \hat{\phi}(\eta; p)], \quad (18)$$

$$(1-p)L_\phi[\hat{\phi}(\eta; p) - \phi_0(\eta)] = p h_\phi \mathbf{N}_\phi[\hat{f}(\eta; p), \hat{\phi}(\eta; p)], \quad (19)$$

$$\hat{f}(0; p) = 0, \hat{f}'(0; p) = 1, \hat{f}'(\infty; p) = \infty, \hat{\phi}(0; p) = 1, \hat{\phi}(\infty; p) = 0, \quad (20)$$

$$\mathbf{N}_f[\hat{f}(\eta; p)] = \frac{\partial^3 \hat{f}}{\partial \eta^3} - 2 \left(\frac{\partial \hat{f}}{\partial \eta} \right)^2 + \hat{f} \frac{\partial^2 \hat{f}}{\partial \eta^2} + \alpha_1 \left[\begin{aligned} & 3 \frac{\partial \hat{f}}{\partial \eta} \frac{\partial^3 \hat{f}}{\partial \eta^3} - \hat{f} \frac{\partial^4 \hat{f}}{\partial \eta^4} \\ & - 2 \eta \frac{\partial^2 \hat{f}}{\partial \eta^2} \frac{\partial^3 \hat{f}}{\partial \eta^3} - 9 \left(\frac{\partial^2 \hat{f}}{\partial \eta^2} \right)^2 \end{aligned} \right]$$

$$- \alpha_2 \left[3 \left(\frac{\partial^2 \hat{f}}{\partial \eta^2} \right)^2 + \eta \frac{\partial^2 \hat{f}}{\partial \eta^2} \frac{\partial^3 \hat{f}}{\partial \eta^3} \right] + 3\beta \left(\frac{\partial^2 \hat{f}}{\partial \eta^2} \right)^2 \frac{\partial^3 \hat{f}}{\partial \eta^3} - Ha^2 \frac{\partial \hat{f}}{\partial \eta}, \quad (21)$$

$$\mathbf{N}_\phi[\hat{\phi}(\eta; p), \hat{f}(\eta; p)] = \frac{\partial^2 \hat{\phi}}{\partial \eta^2} + Sc \hat{f} \frac{\partial \hat{\phi}}{\partial \eta} - Sc \frac{\partial \hat{f}}{\partial \eta} \hat{\phi} - 2Sc\gamma \hat{\phi}, \quad (22)$$

in which p is an embedding parameter and h_f and h_ϕ are the non-zero auxiliary parameters. The nonlinear operators are represented by $\mathbf{N}_f$ and $\mathbf{N}_\phi$.

5. m th-Order Deformation Problem

The resulting problems at m th order deformation are constructed as follows:

$$L_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = h_f \mathbf{R}_f^m(\eta), \quad (23)$$

$$f_m(0) = f'_m(0) = f'_m(\infty) = 0, \quad (24)$$

$$L_\phi[\phi_m(\eta) - \chi_m \phi_{m-1}(\eta)] = h_\phi \mathbf{R}_\phi^m(\eta), \quad (25)$$

$$\phi_m(0) = \phi_m(\infty) = 0, \quad (26)$$

$$\mathbf{R}_f^m(\eta) = f_{m-1}''' - 2 \sum_{k=0}^{m-1} f'_{m-1-k} f'_k + \sum_{k=0}^{m-1} f_{m-1-k} f_k''$$

$$+ \alpha_1 \sum_{k=0}^{m-1} \left(-3 f'_{m-1-k} f_k''' - f_{m-1-k} f_k^{iv} - 2 \eta f_{m-1-k}'' f_k'' - 9 f_{m-1-k}' f_k''' \right)$$

$$+ \alpha_2 \sum_{k=0}^{m-1} \left(3 f_{m-1-k}' f_k'' + \eta f_{m-1-k}' f_k''' \right)$$

$$+ 3\beta \sum_{k=0}^{m-1} f_{k-1}'' \sum_{l=0}^k f_{m-1-k} f_l''' - 2 Ha^2 f_{m-1}'$$

$$\mathbf{R}_\phi^m(\eta) = \phi_{m-1}'' + Sc \sum_{k=0}^{m-1} \phi'_{m-1-k} f_k - Sc \sum_{k=0}^{m-1} f'_{m-1-k} \phi_k - Sc\gamma \phi_{m-1}, \quad (28)$$

$$\chi_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1. \end{cases} \quad (29)$$

The general solutions (f_m, ϕ_m) consisting of special solutions (f_m^*, ϕ_m^*) are

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 e^\eta + C_3 e^{-\eta}, \quad (30)$$

$$\phi_m(\eta) = \phi_m^*(\eta) + C_4 e^\eta + C_5 e^{-\eta}, \quad (31)$$

where C_i ($i = 1 - 5$) indicates the arbitrary constants which are determined by employing the boundary conditions (24) and (26)

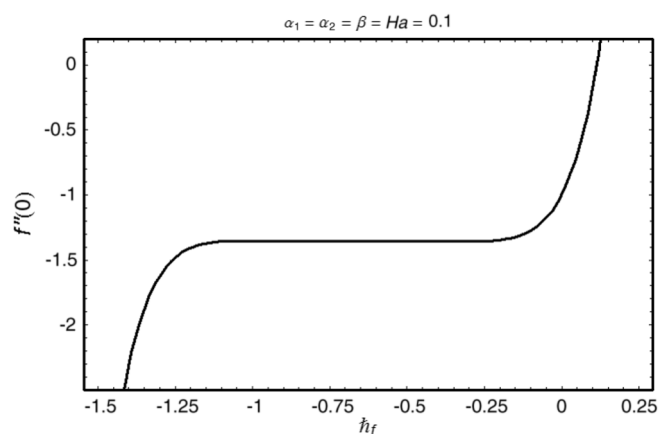
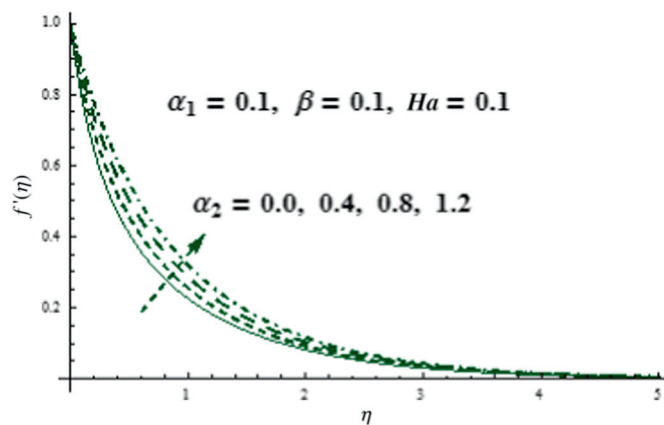
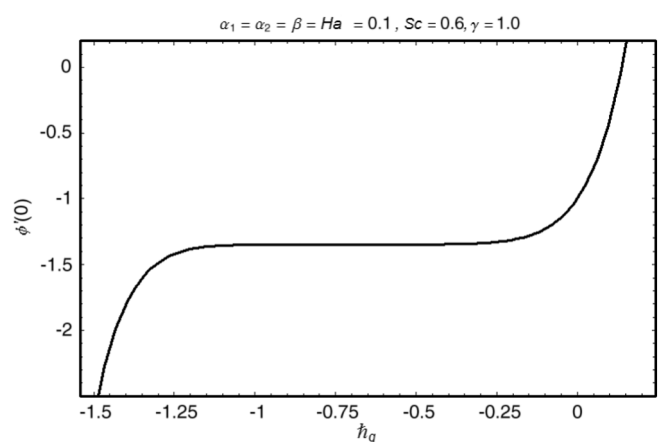
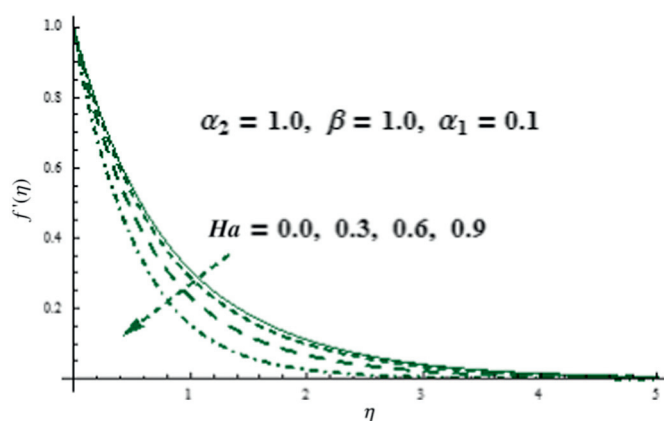
$$C_3 = C_4 = 0, C_2 = \frac{\partial f^*(\eta)}{\partial \eta} \Big|_{\eta=0}, C_1 = -C_2 - f^*(0), C_5 = -\phi_m^*(0) \quad (32)$$

6. Convergence of the Developed Solutions

To construct the series solutions by homotopy analysis technique it is also necessary to check their convergence. Convergence region is the region parallel to h -axis. Hence we have plotted the h -curves of the velocity $f''(0)$ and concentration $\phi'(0)$ in Figs. 1 and 2. Permissible values for the derived solutions are found in the ranges $-1.15 \leq h_f \leq -0.20$ and $-1.15 \leq h_\phi \leq -0.35$.

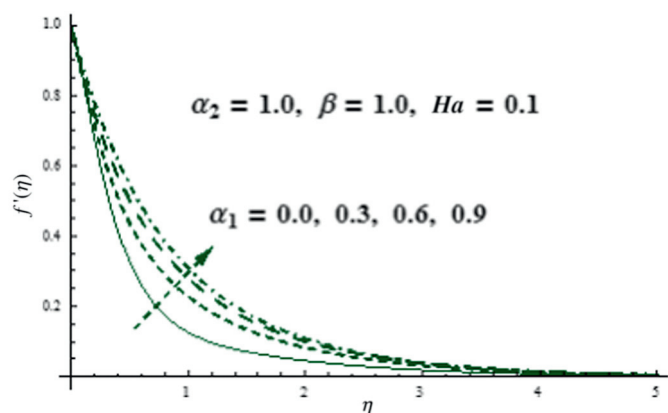
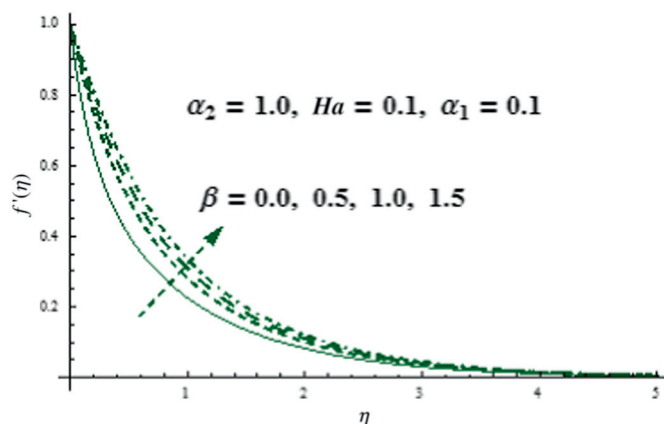
7. Discussion

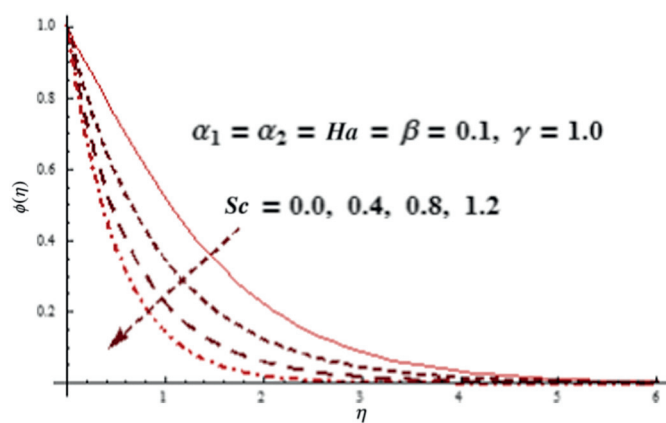
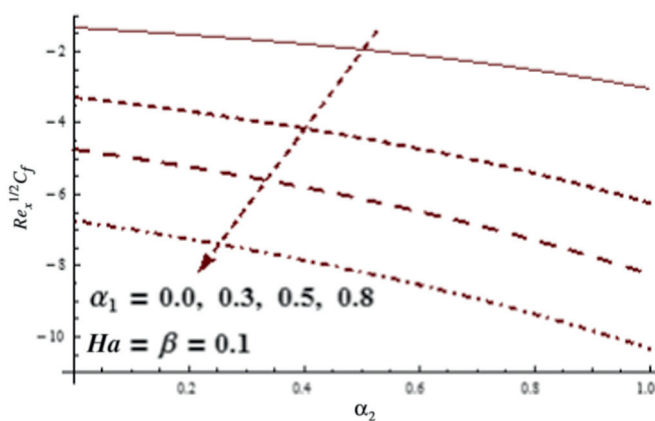
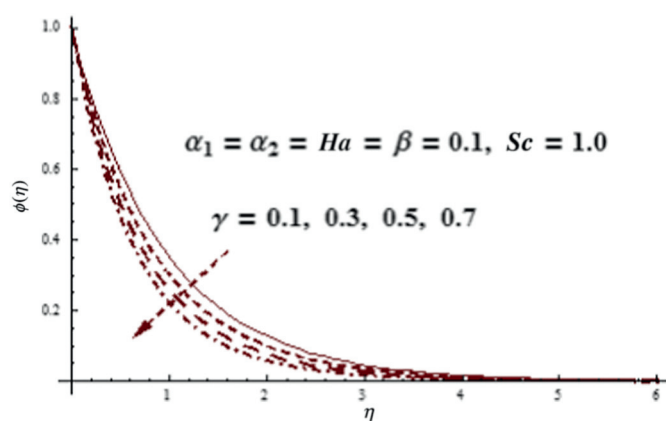
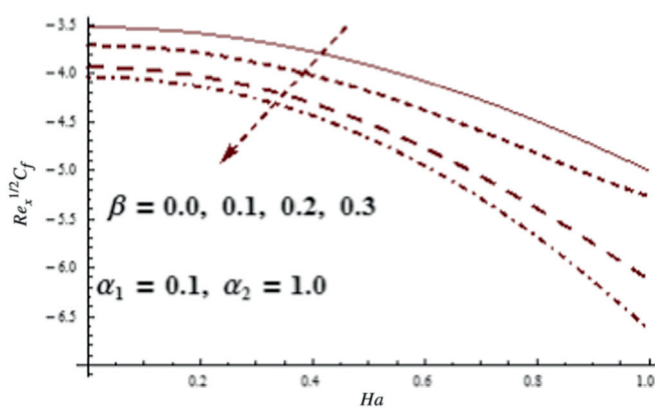
Description of various pertinent parameters on the velocity and concentration distributions is the main motto of this section. Figs. 3 and 4 illustrate the variation of fluid parameters α_1 and α_2 on the velocity profile $f'(\eta)$ respectively. It is observed that the velocity profile $f'(\eta)$ is an increasing function of both α_1 and α_2 . Physically viscosity of the material reduces for larger values of α_1 and α_2 due to which force between the adjacent layers decreases and thus velocity of the fluid increases. Characteristics of Hartman number Ha on velocity distribution is displayed in Fig. 5. It is found that velocity profile decreases via larger Ha . Physically by increasing magnetic field the Lorentz force increases. More resistance is offered to the motion of fluid and thus the velocity of the fluid is reduced. Impact of fluid parameter β on

Fig. 1. h – curve for f .Fig. 4. Impact of α_2 on f' .Fig. 2. h – curve for ϕ .Fig. 5. Impact of Ha on f' .

velocity distribution is depicted in Fig. 6. It is examined that the velocity profile increases near the wall for larger values of β and it vanishes away from the wall. Moreover the momentum boundary layer thickness is also increasing function of β . In fact β is inversely proportional to the viscosity. For larger values of β the viscosity of the fluid decreases and hence the velocity profile enhances. Effect of Schmidt number Sc on concentration distribution is displayed in Fig. 7. Here concentration

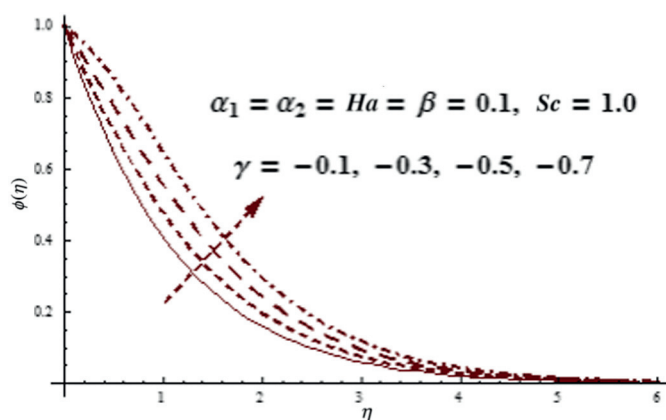
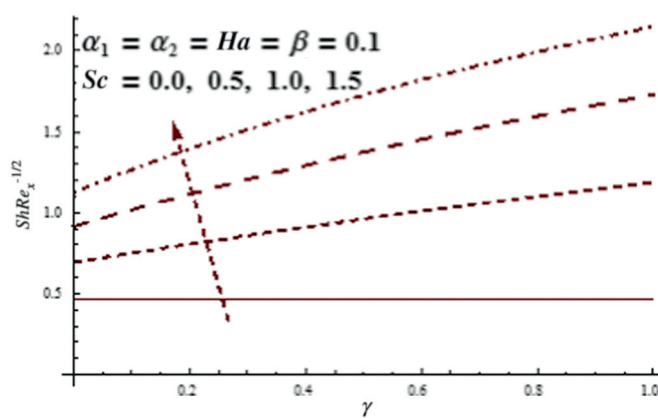
profile and associated boundary layer thickness are decreased when Schmidt number Sc increases. Physically the Schmidt number is dependent on mass diffusion D and an increase in Schmidt number corresponds to a decrease in mass diffusion and the concentration. Fig. 8 depicts the influence of destructive chemical reaction parameter ($\gamma > 0$) on the concentration profile $\phi(\eta)$. It is obvious that the fluid concentration decreases with an increase in the destructive chemical

Fig. 3. Impact of α_1 on f' .Fig. 6. Impact of β on f' .

Fig. 7. Impact of Sc on ϕ .Fig. 10. Impacts of α_1 and α_2 on $Re_x^{1/2}C_f$.Fig. 8. Impact of $\gamma > 0$ on ϕ .Fig. 11. Impacts of β and Ha on $Re_x^{1/2}C_f$.

reaction parameter. In fact higher values of destructive chemical reaction parameter correspond to larger rate of destructive chemical reaction which dissipates or destroys the fluid specie more efficiently. Therefore concentration distribution decreases. Characteristics of generative chemical reaction parameter ($\gamma < 0$) on the concentration profile $\phi(\eta)$ is disclosed through Fig. 9. This figure illustrates that concentration field has an opposite behavior for ($\gamma < 0$) when compared with ($\gamma > 0$).

Physically larger values of generative chemical reaction parameter correspond to higher rate of generative chemical reaction which generates the fluid specie more efficiently and therefore concentration distribution increases. Characteristics of α_1 and α_2 on skin friction are shown in Fig. 10. It is seen that skin friction coefficient increases when α_1 and α_2 are enhanced. Fig. 11 depict the influences of β and Ha on skin friction. It is analyzed that skin friction coefficient increases for larger

Fig. 9. Impact of $\gamma < 0$ on ϕ .Fig. 12. Impacts of Sc and γ on $ShRe_x^{-1/2}$.

β and Ha . Analysis of Sc and γ on local Sherwood number is presented in Fig. 12. This figure shows that local Sherwood number increases for higher values of Sc and γ .

Table 1 shows the convergence of the series solutions for different order of approximations. It is noted that 15th and 20th order of approximations are sufficient for the convergence of momentum and concentration equations respectively. Tables 2 and 3 give the comparison of present results in a limiting sense with the works done by Mukhopadhyay *et al.* [30], Elbashbeshy [54] and Chaudhary *et al.* [55]. It is found that the obtained results in limiting sense are in good agreement.

Table 1

Convergence of homotopy solutions when $\alpha_1 = \alpha_2 = \beta = Ha = 0.1$, $Sc = \gamma = 1.0$ and $h_f = h_b = -0.6$.

Order of approximation	$-f''(0)$	$-\phi'(0)$
1	1.2368	1.4000
5	1.3544	1.7367
10	1.3586	1.7609
11	1.3587	1.7617
15	1.3587	1.7630
17	1.3587	1.7633
20	1.3587	1.7633
30	1.3587	1.7633

Table 2

Comparison for numerical values of $f''(0)$ when $\alpha_1 = \alpha_2 = \beta = Ha = 0$.

$f''(0)$	$f''(0)$	$f''(0)$
Mukhopadhyay <i>et al.</i> [30] –1.281812	Elbashbeshy [54] –1.28181	Present –1.2818

Table 3

Comparison for numerical values of $f''(0)$ for Ha when $\alpha_1 = \alpha_2 = \beta = 0$.

Ha^2	$f''(0)$	$f''(0)$
0.0	Chaudhary <i>et al.</i> [55] –1.2821	Present –1.2818
0.2		–1.3133
0.4		–1.4030

8. Concluding Remarks

We have investigated the characteristics of free convection boundary layer flow of third grade fluid induced by exponentially stretching sheet. The present analysis leads to the following observations.

- Influences of α_1 , α_2 and β on velocity distribution are similar in a qualitative manner.
- Magnetic field retards the fluid velocity and momentum boundary layer thickness.
- Concentration distribution has opposite results for destructive ($\gamma > 0$) and generative ($\gamma < 0$) chemical reactions.
- An increase in Schmidt number Sc causes a decrease in the concentration distribution and the boundary layer thickness.
- Impacts of α_1 , α_2 , β and Ha on skin friction are similar in a qualitative sense.
- Local Sherwood number enhances for larger Sc and γ .

References

- [1] S. Abelman, E. Momoniat, T. Hayat, Couette flow of a third grade fluid with rotating frame and slip condition, *Nonlinear Anal. Real World Appl.* 10 (2009) 3329–3334.
- [2] T. Hayat, A. Shafiq, A. Alsaedi, Effect of joule heating and thermal radiation in flow of third grade fluid over radiative surface, *PLoS One* 9 (2014) e83153.
- [3] S. Abbasbandy, T. Hayat, On series solution for unsteady boundary layer equations in a special third grade fluid, *Commun. Nonlinear Sci. Numer. Simul.* 16 (2011) 3140–3146.
- [4] S.A. Shehzad, T. Hussain, T. Hayat, M. Ramzan, A. Alsaedi, Boundary layer flow of third grade nanofluid with Newtonian heating and viscous dissipation, *J. Cent. South Univ.* 22 (2015) 360–367.
- [5] T. Hayat, A. Shafiq, A. Alsaedi, S. Asghar, Effect of inclined magnetic field in flow of third grade fluid with variable thermal conductivity, *AIP Adv.* 5 (2015) 087108.
- [6] K. Bhattacharyya, Steady boundary layer flow and reactive mass transfer past an exponentially stretching sheet in an exponentially moving free stream, *J. Egypt. Math. Soc.* 20 (2012) 223–228.
- [7] S. Mukhopadhyay, R.S.R. Gorla, Diffusion of chemically reactive species of a Casson fluid flow over an exponentially stretching surface, *Therm. Energy Power Eng.* 3 (2014) 216–221.
- [8] M.S. Alhuthali, S.A. Shehzad, H. Malaikah, T. Hayat, Three dimensional flow of viscoelastic fluid by an exponentially stretching surface with mass transfer, *J. Pet. Sci. Eng.* 119 (2014) 221–226.
- [9] S. Mukhopadhyay, K. Bhattacharyya, G.C. Layek, Mass transfer over an exponentially stretching porous sheet embedded in a stratified medium, *Chem. Eng. Commun.* 201 (2014) 272–286.
- [10] T. Hayat, S. Asad, M. Mustafa, A. Alsaedi, MHD stagnation-point flow of Jeffrey fluid over a convectively heated stretching sheet, *Comput. Fluids* 108 (2015) 179–185.
- [11] T. Hayat, T. Muhammad, S.A. Shehzad, A. Alsaedi, Soret and Dufour effects in three-dimensional flow over an exponentially stretching surface with porous medium, chemical reaction and heat source/sink, *Int. J. Numer. Methods Heat Fluid Flow* 25 (2015) 762–781.
- [12] K. Bhattacharyya, G.C. Layek, Chemically reactive solute distribution in MHD boundary layer flow over a permeable stretching sheet with suction or blowing, *Chem. Eng. Commun.* 197 (2010) 1527–1540.
- [13] K. Bhattacharyya, G.C. Layek, Slip effects on diffusion of chemically reactive species in boundary layer flow due to a vertical stretching sheet with suction or blowing, *Chem. Eng. Commun.* 198 (2011) 1354–1365.
- [14] F. Mabood, W.A. Khan, A.I.M. Ismail, MHD stagnation point flow and heat transfer impinging on stretching sheet with chemical reaction and transpiration, *Chem. Eng. J.* 273 (2015) 430–437.
- [15] M. Sheikh, Z. Abbas, Effects of thermophoresis and heat generation/absorption on MHD flow due to an oscillatory stretching sheet with chemically reactive species, *J. Magn. Magn. Mater.* 396 (2015) 204–213.
- [16] G. Swapna, L. Kumar, P. Rana, B. Singh, Finite element modeling of a double-diffusive mixed convection flow of a chemically-reacting magneto-micropolar fluid with convective boundary condition, *J. Taiwan Inst. Chem. Eng.* 47 (2015) 18–27.
- [17] T. Hayat, Z. Hussain, A. Alsaedi, S. Asghar, Carbon nanotubes effects in the stagnation point flow towards a nonlinear stretching sheet with variable thickness, *Adv. Powder Tech.* (2016), <http://dx.doi.org/10.1016/j.appt.2016.06.001>.
- [18] C. Zhang, L. Zheng, X. Zhang, G. Chen, MHD flow and radiation heat transfer of nanofluids in porous media with variable surface heat flux and chemical reaction, *Appl. Math. Model.* 39 (2015) 165–181.
- [19] D. Mythili, R. Sivaraj, Influence of higher order chemical reaction and non-uniform heat source/sink on Casson fluid flow over a vertical cone and flat plate, *J. Mol. Liq.* 216 (2016) 466–475.
- [20] P.V.S. Narayana, D.H. Babu, Numerical study of MHD heat and mass transfer of a Jeffrey fluid over a stretching sheet with chemical reaction and thermal radiation, *J. Taiwan Inst. Chem. Eng.* 59 (2016) 18–25.
- [21] T. Hayat, M.I. Khan, M. Farooq, A. Alsaedi, M. Waqas, T. Yasmeen, Impact of Cattaneo–Christov heat flux model in flow of variable thermal conductivity fluid over a variable thicked surface, *Int. J. Heat Mass Transf.* 99 (2016) 702–710.
- [22] T. Hayat, M.I. Khan, M. Farooq, T. Yasmeen, A. Alsaedi, Stagnation point flow with Cattaneo–Christov heat flux and homogeneous–heterogeneous reactions, *J. Mol. Liq.* 220 (2016) 49–55.
- [23] M. Imtiaz, T. Hayat, A. Alsaedi, A. Hobiny, Homogeneous–heterogeneous reactions in MHD flow due to an unsteady curved stretching surface, *J. Mol. Liq.* 221 (2016) 245–253.
- [24] T. Hayat, S. Qayyum, M. Imtiaz, A. Alsaedi, Impact of Cattaneo–Christov heat flux in Jeffrey fluid flow with homogeneous–heterogeneous reactions, *PLoS One* 11 (2016) e0148662.
- [25] G.R. Machireddy, Chemically reactive species and radiation effects on MHD convective flow past a moving vertical cylinder, *Ain Shams Eng. J.* 4 (2013) 879–888.
- [26] Z. Ziaabakhsh, G. Domairry, H. Bararnia, H. Babazadeh, Analytical solution of flow and diffusion of chemically reactive species over a nonlinearly stretching sheet immersed in a porous medium, *J. Taiwan Inst. Chem. Eng.* 41 (2010) 22–28.
- [27] K. Bhattacharyya, Dual solutions in boundary layer stagnation-point flow and mass transfer with chemical reaction past a stretching/shrinking sheet, *Int. Commun. Heat Mass Transfer* 38 (2011) 917–922.
- [28] S. Chen, Q. Sun, Numerical study of shock/boundary layer interaction of chemically reacting flow in shock tube, *Procedia Eng.* 126 (2015) 617–621.
- [29] M.C. Raju, N.A. Reddy, S.V.K. Varma, Analytical study of MHD free convective, dissipative boundary layer flow past a porous vertical surface in the presence of thermal radiation, chemical reaction and constant suction, *Ain Shams Eng. J.* 5 (2014) 1361–1369.
- [30] S. Mukhopadhyay, I.C. Moindal, T. Hayat, MHD boundary layer flow of Casson fluid passing through an exponentially stretching permeable surface with thermal radiation, *Chin. Phys. B* 23 (2014) 104701.
- [31] T. Hayat, M. Imtiaz, A. Alsaedi, Impact of magnetohydrodynamics in bidirectional flow of nanofluid subject to second order slip velocity and homogeneous–heterogeneous reactions, *J. Magn. Magn. Mater.* 395 (2015) 294–302.
- [32] Y. Lin, L. Zheng, X. Zhang, L. Ma, G. Chen, MHD pseudo-plastic nano fluid unsteady flow and heat transfer in a finite thin film over stretching surface with internal heat generation, *Int. J. Heat Mass Transf.* 84 (2015) 903–911.

- [33] M. Sheikholeslami, D.D. Ganji, M.Y. Javed, R. Ellahi, Effect of thermal radiation on magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model, *J. Magn. Magn. Mater.* 374 (2015) 36–43.
- [34] U. Farooq, Y.L. Zhao, T. Hayat, A. Alsaedi, S.J. Liao, Application of the HAM-based Mathematica package BVP4 on MHD Falkner–Skan flow of nanofluid, *Comput. Fluids* 11 (2015) 69–75.
- [35] S.A. Shehzad, Z. Abdullah, A. Alsaedi, F.M. Abbaasi, T. Hayat, Thermally radiative three-dimensional flow of Jeffrey nanofluid with internal heat generation and magnetic field, *J. Magn. Magn. Mater.* 397 (2016) 108–114.
- [36] M. Turkyilmazoglu, Solution of Thomas–Fermi equation with a convergent approach, *Commun. Nonlinear Sci. Numer. Simul.* 17 (2012) 4097–4103.
- [37] S. Han, L. Zheng, C. Li, X. Zhang, Coupled flow and heat transfer in viscoelastic fluid with Cattaneo–Christov heat flux model, *Appl. Math. Lett.* 38 (2014) 87–93.
- [38] M. Sheikholeslami, M. Hatami, D.D. Ganji, Micropolar fluid flow and heat transfer in a permeable channel using analytical method, *J. Mol. Liq.* 194 (2014) 30–36.
- [39] L. Zheng, C. Zhang, X. Zhang, J. Zhang, Flow and radiation heat transfer of a nanofluid over a stretching sheet with velocity slip and temperature jump in porous medium, *J. Frankl. Inst.* 350 (2013) 990–1007.
- [40] F.M. Abbasi, S.A. Shehzad, T. Hayat, A. Alsaedi, M.A. Obid, Influence of heat and mass flux conditions in hydromagnetic flow of Jeffrey nanofluid, *AIP Adv.* 5 (2015) 037111.
- [41] T. Hayat, M. Waqas, S.A. Shehzad, A. Alsaedi, MHD stagnation point flow of Jeffrey fluid by a radially stretching surface with viscous dissipation and joule heating, *J. Hydrol. Hydromech.* 63 (2015) 311–317.
- [42] U. Farooq, T. Hayat, A. Alsaedi, S. Liao, Heat and mass transfer of two-layer flows of third-grade nano-fluids in a vertical channel, *Appl. Math. Comput.* 242 (2014) 528–540.
- [43] S. Abbasbandy, T. Hayat, A. Alsaedi, M.M. Rashidi, Numerical and analytical solutions for Falkner–Skan flow of MHD Oldroyd-B fluid, *Int. J. Numer. Methods Heat Fluid Flow* 24 (2014) 390–401.
- [44] T. Hayat, M. Farooq, A. Alsaedi, F. Al-Solamy, Impact of Cattaneo–Christov heat flux in the flow over a stretching sheet with variable thickness, *AIP Adv.* 5 (2015) 087159.
- [45] T. Hayat, M. Waqas, S.A. Shehzad, A. Alsaedi, Effects of joule heating and thermophoresis on stretched flow with convective boundary conditions, *Sci. Iran. B* 21 (2014) 682–692.
- [46] S.A. Shehzad, M. Waqas, A. Alsaedi, A. Alsaedi, Flow and heat transfer over an unsteady stretching sheet in a micropolar fluid with convective boundary condition, *J. Appl. Fluid Mech.* 9 (2016) 1437–1445.
- [47] M. Waqas, T. Hayat, M. Farooq, S.A. Shehzad, A. Alsaedi, Cattaneo–Christov heat flux model for flow of variable thermal conductivity generalized Burgers fluid, *J. Mol. Liq.* 220 (2016) 642–648.
- [48] M. Khan, W.A. Khan, Three-dimensional flow and heat transfer to Burgers fluid using Cattaneo–Christov heat flux, *J. Mol. Liq.* 221 (2016) 651–657.
- [49] T. Hayat, M. Waqas, S.A. Shehzad, A. Alsaedi, Mixed convection flow of viscoelastic nanofluid by a cylinder with variable thermal conductivity and heat source/sink, *Int. J. Numer. Methods Heat Fluid Flow* 26 (2016) 214–234.
- [50] M. Khan, W.A. Khan, A.S. Alshomrani, Non-linear radiative flow of three-dimensional Burgers nanofluid with new mass flux effect, *Int. J. Heat Mass Transf.* 101 (2016) 570–576.
- [51] M. Waqas, M.I. Khan, M. Farooq, A. Alsaedi, T. Hayat, T. Yasmeen, Magnetohydrodynamic (MHD) mixed convection flow of micropolar liquid due to nonlinear stretched sheet with convective condition, *Int. J. Heat Mass Transf.* 102 (2016) 766–772.
- [52] M.I. Khan, M.Z. Kiyani, M.Y. Malik, T. Yasmeen, M.W.A. Khan, T. Abbas, Numerical investigation of magnetohydrodynamic stagnation point flow with variable properties, *Alex. Eng. J.* (2016), <http://dx.doi.org/10.1016/j.aej.2016.04.037>.
- [53] M. Farooq, M.I. Khan, M. Waqas, T. Hayat, A. Alsaedi, M.I. Khan, MHD stagnation point flow of viscoelastic nanofluid with non-linear radiation effects, *J. Mol. Liq.* 221 (2016) 1097–1103.
- [54] E.M.A. Elbashbeshy, Heat transfer over an exponentially stretching continuous surface with suction, *Arch. Mech.* 53 (2001) 643–651.
- [55] S. Chaudhary, S. Singh, S. Chaudhary, Thermal radiation effects on MHD boundary layer flow over an exponentially stretching surface, *Appl. Math.* 6 (2015) 295–303.